A Probabilistic Approach to Determine Uncertainty in Calculated Water Saturation

M.C. Bowers and D.E. Fitz, ExxonMobil Exploration Company

This paper was originally presented at the SPWLA 41st Annual Logging Symposium, June 4-7, 2000.

Abstract

Uncertainty in calculated water saturation has a direct economic impact on both exploration and development projects, yet is rarely quantified by petrophysicists. Quantifying the sensitivity associated with each parameter in any water saturation model is required to determine the most cost-effective way to reduce the total uncertainty to an acceptable level and provides the framework for doing value of information calculations. An analytic method to estimate uncertainty in water saturation using the Dual Water model is presented. The method is based on the general formula for error propagation and implicit differentiation to calculate the standard deviation in water saturation from which the 80% confidence level (P_{10} and P_{90}) is derived. Normal distributions are used to characterize the uncertainty for all log and model parameters. The approach is compared to Monte Carlo methods to assess the advantages and limitations of an analytic approach and the assumptions of normal distributions. An example is presented illustrating how this approach can be used to determine which types of petrophysical measurements provide the greatest reduction in uncertainty for a given cost. The method can be extended to any water saturation model.

Introduction

The uncertainty associated with any water saturation calculation depends on the accuracy and precision of the log and core measurements. The uncertainty in water saturation has a direct impact on the economics of any project but petrophysicists rarely quantify uncertainty.

The products from most petrophysical analyses are used, with other data, to determine the monetary value of a prospect or project. Figure 1 is an influence diagram that illustrates the variables that impact estimations of net present value, or expected monetary value, for a project (Bouchard and Fox, 1999). From Figure 1, we see that estimations of oil saturation, irreducible water saturation and porosity are key components of those calculations. If we want to quantify the uncertainty in net present value, we must first quantify the uncertainty in all of the variables in Figure 1. As petrophysicists, we are responsible for quantifying the uncertainty water saturation and porosity.

The Dual Water Model (DWM) is commonly used to calculate water saturation (Clavier et al, 1984). Parameters are derived directly from logging measurements, core measurements and interpretation models. In a probabilistic approach, the uncertainties in all parameters must be considered.

We can think of uncertainty as meaning any measurement or knowledge of any parameter is imperfect. We only know the value of a measurement to within certain limits. In terms of DWM, each log, core and interpretation based parameter is associated with a distribution of possible values at each depth. Therefore, the water saturation at each depth is associated with a distribution of values at each depth.

By understanding what causes uncertainty, it can be quantified and reduced by acquiring only those additional measurements associated with the critical parameters needed to reduce the uncertainty to an acceptable level. Money should not be spent to acquire data that does not help reduce uncertainty or spend time on analysis techniques that do not reduce uncertainty.

In the following text, a method of implementing a probabilistic calculation of water saturation derived from an analytic approach is illustrated. An example is used to illustrate how to implement the methodology and the accuracy of that method as compared to a Monte Carlo simulation. Reasons for using an analytic solution rather than a Monte Carlo solution are also discussed.

Solution Constraints

The method chosen to calculate the uncertainty must meet a number of requirements. These requirements are:

1. Uncertainty in all of the variables must be considered because the sensitivity to a particular parameter cannot be determined apriori.
2. The method for calculating uncertainty must be objective and reproducible. Subjective methods are not reproducible.
3. The method must be simple and relatively quick to implement.
4. The method must be implemented in the user defined programming modules found in currently available software packages. Efficiency is greatly reduced if the data must be transferred between software packages.
5. The method must also be free of any models. Model assumptions could be a bigger source of uncertainty than the uncertainties associated with the variables.
6. The method must be flexible enough for dealing with different types and qualities of data.
7. The method must produce a set of parameters to describe uncertainty that is compatible with other disciplines. The method must produce P_{10}, P_{50} and P_{90} estimates of water saturation or the most likely estimate and the 80% confidence interval.
8. The method must produce a set of error curves for water saturation. These curves are a necessary tool to compare the level of certainty from depth to depth or zone to zone.

A Probabilistic Approach

Monte Carlo Models. Monte Carlo simulation is commonly used to implement a probabilistic solution where the input parameters are defined in terms of distributions. Random samples are generated from each distribution and used in an interpretation model to generate an outcome or realization. The process is iterated many times and eventually stopped when some predefined criteria are met (Voss, 1996).

Latin Hypercube Models. The most common alternative is Latin Hypercube. The individual distributions are divided into *n* intervals of equal probability where *n* is the number of iterations to be performed. The first interval to be sampled is picked using a random number. A second random number is used to determine where within that interval a sample should be derived. As the process is repeated, an interval will not be resampled (Voss, 1996). Because of the sampling methodology, the Latin Hypercube technique results in a complete sampling of the input distributions even when the number of iterations is relatively low.

The problem with implementing these simulations is that special software is needed which is not currently found in commercially available log analysis software packages. The implementation of these models over a large section of log data can also be very time consuming. An advantage is that any type of distribution can be used to characterize the uncertainty distribution of any parameter.

Analytic Solutions. An alternative to a simulation model is an analytic solution derived using the General Formula for Error Propagation (Taylor 1997). These solutions produce the same results as a simulation model if normal distributions are used to describe the uncertainty associated with each of the parameters.

The standard deviation of *y*, which is any function of *x,…z*, can be calculated using (Taylor, 1997 and Chen and Fang, 1986):

$$\sigma_y^2 = \sum_{i=1}^{n}\left(\frac{\partial y}{\partial x_i}\sigma_{x_i}\right)^2 \quad (1)$$

Freedman and Ausburn (1985) defined the best estimate of the uncertainty associated with each independent variable as:

$$\sigma_{x_i} = (\pm q)x_i \quad (2)$$

where x is the best estimate of parameter i and q is the estimate of uncertainty of x expressed as a ratio.

Because normal distributions are assumed, the normal error intergral can be used to derive any probability of occurrence (Taylor, 1997). P10 and P90 estimates are defined with the following:

$$P_{10}, P_{90} = P_{50} \pm 0.8225\,\sigma_y \quad (3)$$

Because the total uncertainty of the model is the summation of the uncertainties associated with each model, the sensitivity of the model to each parameter is known. Understanding the sensitivity of the model is useful for understanding what, if any, additional data are needed to achieve an acceptable level of uncertainty.

There are some very important assumptions that must be understood when applying an analytic solution. The errors associated with each of the parameters must be random and independent. If the errors are not, the total uncertainty will be underestimated. In addition, the errors cannot be systematic.

An Analytic Solution for the Dual Water Model

Chen and Fang (1986) derived analytic solutions for uncertainty in water saturation calculated from Archie's Equation. However, an analytic solution for DWM is more difficult to derive. DWM has the following form:

$$C_T = \frac{Sw_T^n \phi_T^m}{a}\left[C_{wf} + \frac{Sw_b}{Sw_T}\left(C_{wb} - C_{wf}\right)\right] \quad (4)$$

where:

C_t =total conductivity

Sw_T =total water saturation

C_{wf} =conductivity formation water

C_{wb} =conductivity bound water

Sw_b =bound water saturation

m =cementation exponent

n =saturation exponent

a =tortuosity factor

ϕ_t =total porosity

An estimation of Sw_b is made using:

$$Sw_b = \frac{\phi_{sh}^t V_{sh}}{\phi_e^t} \quad (5)$$

where:

ϕ_{sh} =shale porosity

V_{sh} =shale volume

ϕ_e =effective porosity

Equation 4 cannot be solved explicitly for Sw_T (unless n=2) and therefore deriving the partial derivatives with respect to Sw_T requires implicit differentiation (see Johnson and Kiokemeister; 1959). The analytic solution for DWM must have the following form:

$$\sigma^2_{Sw_t} = \sum_{i=1}^{n} \left(\frac{\partial Sw_t}{\partial x_i} \right)^2 \sigma^2_{x_i} \quad (6)$$

Through substitution and simplification, Equation 4 can be rewritten as:

$$f = \phi_T \phi_T Sw_t C_{wf} + \phi_T^{m-1} Sw_T^{n-1} V_{sh} \phi_{sh} (C_{wb} - C_{wf}) - aC_T = 0 \quad (7)$$

The partial derivative of Sw_T with respect to each variable is found from the rearrangement of Equation 7 and the application of Equation 8:

$$\frac{\partial Sw_T}{\partial x_i} = - \frac{\frac{\partial f}{\partial x_i}}{\frac{\partial f}{\partial Sw_T}} \quad (8)$$

where:

xi =a, m, n, f sh, f t, Vsh, Cwb, Cwf, Swb

Substitution of Equation 8 into Equation 6 produces an estimate of uncertainty in water saturation. Equation 3 is used to estimate the 80% confidence interval. All of the partial derivatives of f with respect each of the variables are found in Appendix 1.

The assumptions associated with an analytic solution need further discussion. All logging tools make measurements of overlapping volumes as they are moved up the borehole. Therefore, the measurements are not completely random but probably do not significantly impact the calculation of uncertainty. Additionally, logging measurement errors can be systematic when boreholes become enlarged or rugose. Under these circumstances, the use of tools that are greatly affected by borehole conditions must be avoided. We will also assume that the application of DWM results in an accurate estimate of water saturation if there were no measurement errors (i.e. model error is small). Finally we must assume there are no systematic errors in the logging or core measurements.

Estimating Parameter Uncertainty

The most difficult part of undertaking a probabilistic approach is attempting to quantify the uncertainty in each of the parameters. For parameters estimated directly from logging measurements, the best estimate is the log measurement at each depth and the uncertainty estimated from the tool specifications. For example, the conductivity derived from an induction tool has an uncertainty of 0.75 mmho.

Most parameters are estimated from a transform or log response model and therefore need additional consideration. For example, porosity estimates derived from bulk density. The uncertainty in the logging measurement and the matrix and fluid parameters must be considered. In general, the uncertainty derived for the any porosity estimate is at least 1.5 p.u.. If core porosity is available, the comparison between core porosity and log derived porosity can be used to determine the uncertainty of the log derived porosity. For empirically derived transforms, the uncertainty of the model must also be considered.

Special consideration must be given to shale volume uncertainty estimates. These estimates are also derived from logging measurements and an interpretation model. However, the shale end-members (clean sand and shale) can usually be defined with more certainty than the intermediate rock types. As a result, the uncertainties in shale volume can be varied as a function of the shale volume estimate. Additional care must be exercised because the differences due to model selection could be large and systematic (i.e. use of the Clavier model when a linear model is more appropriate). For most cases, the uncertainty in shale volume is usually between 0.05 and 0.20.

The uncertainty in electrical properties is also difficult to estimate. In most analyses, single estimates of "m" and "n" are made for an interval or all of the sands. Many assume that the range in these parameters is very small and are correlated with depth. However, the number of samples is usually limited. For example, if the range in "m" was estimated to be 1.8 to 2.0, with a most likely of 1.9 at depth z, the probability that the actual value at depth z+1 being 1.8 or 2.0 is extremely small. The actual value at z+1 is most likely very close to the value at depth z. If there is a correlation with depth, we are calculating the bias caused by variance in these parameters.

These problems are minimized if there is data available to constrain the ranges or different ranges are assigned to different intervals or facies. For example, the range for a turbidite would be different than the range for a debrite (if the different rock types are identified with certainty). Under these conditions, the problem is an uncertainty calculation rather than a bias problem.

The biggest problem with estimating the range of uncertainty associated with electrical properties is that most data sets contain only a few measurements. A number of proprietary data sets (with 100+ samples per well) illustrate that the range in "m" over a very small depth range is as large as the range over the entire well and there is no correlation with depth. To illustrate these observations, the variance in "m" was calculated for a well, for samples from small depth intervals and for all samples. These calculations are listed in Table 1.

The overall range in "m" is very large and we speculate that the range in "n" is also very large. In most cases, an initial analysis is done with little or no available information for the estimation of uncertainty in parameters. Therefore, the estimates of uncertainty in these values should be large.

Voss (1998) presented a number of comments to aid in the estimation of ranges. First, a single interpreter should avoid making estimates on their own. A single interpreter often lacks the needed knowledge to correctly estimate every parameter. In addition, many interpreters have a bias that smaller errors are better and they appear more knowledgeable about the subject. The error must reflect the level of knowledge about the parameters and the data quality. A standard set of uncertainty ranges must be avoided because there is no standard situation in which to apply them.

Unusual events also pose a special problem. Most people have a better recall of unusual events and therefore have a tendency to overestimate the probability of such an event occurring, especially if that event occurred recently (Voss, 1998). Another very common mistake is to allow a very small amount of data to quantify the range of uncertainty for a particular parameter. If data sets are small, the ranges probably need to be increased.

The final and probably most difficult problem to overcome is the culture and preconceived ideas of an organization. Methods and ranges of uncertainty applied to any analysis must be questioned every time they are applied.

Boundary Conditions

The fact that water saturation must lie between zero and one must be considered. Calculations using ranges of uncertainties will result in estimates of water saturation outside those limits. Therefore, if the P_{10} and P_{90} estimates of water saturation are less than 0 or greater than 1, they must be set equal to 0 or 1. If the saturation values are too large or too small, the "best guesses" and ranges must be reconsidered and calculations remade.

Validation of the Analytic Solution for DWM

As part of the validation process, the final water saturation distributions from the Monte Carlo simulations for DWM were checked to make sure they are normal. Figure 2 is an example of the distributions of water saturation produced from DWM and a Monte Carlo simulation when normal distributions are used to characterize the uncertainty in the input variables. The final distributions are normal. Figure 2 also contains an example of the distribution of water saturation values when uniform distributions are used to characterize the uncertainty in input variables.

Comparisons of P_{10} and P_{90} estimates of water saturation for thirty-four cases derived from the analytic and Monte Carlo methods were also made. Obviously, the P_{10} and P_{90} values derived from an analytic solution must be equal to the PP_{10} and P_{90} values derived from a Monte Carlo simulation. Figure 3 is an illustration of the relationships between the P_{10} and P_{90} values derived from both methods. The relationships are excellent and validate the analytic solution.

Other comparisons were also made. Figure 4 is an illustration of the comparisons between the P_{10} and P_{90} estimates derived from analytic solutions and P_{10} and P_{90} estimates derived from Monte Carlo simulations based on uniform distributions to characterize the uncertainties in the input variables. The difference between estimates increases as water saturation increases. These cases were run to illustrate the potential differences associated with an analytic solution if the distributions are not normal. As water saturation increases, the analytic solution tends to produce larger estimates of uncertainty than the Monte Carlo simulation.

Sensitivity Calculations

Sensitivity of DWM relative to any variable is proportional to the contribution to total variance associated with each variable. The first example was calculated for the Archie Equation using an analytic solution derived by Chen and Fang (1986). The variance estimates were made using a formation resistivity value of 50 ohmm, a saturation exponent of 1.8, a cementation exponent of 1.7, and a formation water resistivity of 0.064 at 120$_o$F. Porosity was varied from 0.01 to 0.50. The level of uncertainty for all of the variables was estimated at 0.10.

The results are illustrated in Figure 5. The cementation exponent is the most important parameter at all porosity values less than 37%. The sensitivity to porosity decreases as porosity increases.

A similar set of sensitivities were calculated using DWM. The same parameters used for the Archie example are used here, except a shale volume of 20% with a resistivity of 0.7 ohmm and a shale porosity of 28% was assumed. The most important parameters for the majority of porosity values are the saturation and cementation exponents (Figure 6). As porosity increases, the importance of the saturation exponent increases.

In both examples, as water saturation decreases, the total variance decreases and therefore, the total uncertainty in water saturation decreases. An equally important observation is that the range of uncertainty in water saturation decreases as the standard deviation, or uncertainty, associated with the parameters decreases. Uncertainty is most critical to evaluate at intermediate and high water saturation.

The main reason for performing the sensitivity calculations is to determine the relative value of reducing the uncertainty associated with each variable. For the previously described cases, knowing the value of the cementation and saturation exponents with as much certainty as possible is most important for obtaining the best estimate of water saturation. These types of calculations can be made at many times during a project.

The first time these calculations can be made is when the well is being planned. For a particular geologic setting, the analyst can use a set of likely values and ranges to determine what types and quantity of data should be collected to produce the least amount of uncertainty. The same sensitivity calculations are also useful to determine if core is needed for formation evaluation purposes, and if so, what types of core measurements should be made. The design of the data acquisition program should be influenced by the level of certainty needed in water saturation and designed to reduce uncertainty in the most cost-effective manner. Sensitivity calculations can also be made when the initial log analysis is completed. The sensitivity and total uncertainty calculations can be used to choose the locations for core samples and the types of measurements needed to reduce the largest amount of uncertainty. Sensitivity calculations are also useful to determine if more advanced petrophysical analysis techniques should be applied.

An Example

A synthetic data set was created to illustrate the application of a probabilistic approach and to illustrate the validation of the approach (see Appendix 2 for a description of how the example was constructed). Because a synthetic data set was used, the true answer is known. The objective of this example is to determine if the known answer falls within the P_{10} and P_{90} solutions derived from an analytic approach without any knowledge of the core based parameters. Figure 7 is a plot of the synthetic data used to calculate water saturation.

Figure 8 is log plot of the probabilistic solution for water saturation. The cementation and saturation exponents were estimated at 1.85 and 2.0 respectively with an uncertainty of 0.15. Shale volume was estimated using a linear model and uncertainty was varied as a function of shale volume. If shale volume was initially estimated at less than 20%, the level of uncertainty was 0.05. If shale volume was initially estimated at greater than 20%, the uncertainty was estimated at 0.15. Porosity was calculated from the bulk density and uncertainty in porosity was estimated at 0.10 or 1.5 p.u. units, whichever is greater. The uncertainty in resistivity was estimated at 0.10 or 0.75 mmho, whatever is greater. The uncertainty in all other parameters was estimated at 0.10. Also illustrated in Figure 9, is the known answer. As illustrated the known answer falls within the P_{10} and P_{90} estimates.

The mean saturation and cementation exponents of 1.8 and 1.9 respectively, with a standard deviation of 0.12. Accurate estimation of these parameters is dependent upon the number of samples. Figure 9 is a plot of the standard deviation in cementation exponent versus the number of samples used in the calculation. The samples were randomly ordered and the standard deviation continuously calculated.

The estimation of standard deviation (variance squared) does not become stable until the number of samples is larger than about 50. Two interpretations can be made from an examination of these plots. First, a large number of samples are necessary to get an accurate estimate of variance associated with cementation exponents and probably saturation exponents. Second, if a large number of samples are not available, estimating the variance is difficult and it is probably best to find analogous data sets.

Two groups of clean sandstones from the Hill and Milburn (1956) data set were also examined for the magnitude in "m". Histograms for each clean sandstone groups are found in Figure 10 and Figure 11. Both groups have about the same average "m" value but very different standard deviations (0.086 and 0.230). These examples illustrate the difficulty in estimating the magnitude of the uncertainty in '"m" and "n".

Discussion and Conclusions

A probabilistic solution for water saturation must be an important part of any petrophysical analysis. The best method for implementing a probabilistic solution is a Monte Carlo simulation. However, these solutions are often impractical to implement because special software is needed and a considerable amount of time is usually needed for implementation. An excellent alternative is an analytic solution. These solutions can be implemented in any user defined programming module found in any commercially available log analysis software package. Most important, analytic solutions are quick and easy to implement and provide an accurate answer.

A probabilistic solution also provides a significant amount of information. The P_{10} and P_{90} curves can be presented to geologists and reservoir engineers as a quantitative measure of how accurate the estimation of water saturation can be given a certain level of knowledge. Sensitivity calculations are needed to determine the best and most cost effective method to further reducing uncertainty.

The sensitivity calculations are also useful for getting a quantitative understanding where more effort and resources should be used to reduce the uncertainty in water saturation. For example, if an analysis is completed before core plugs are taken, the P_{10} and P_{90} curves can be used to determine where the data should be collected so that the largest amount of uncertainty can be eliminated. The sensitivity calculations provide the means to determine what types of measurements must be made to reduce uncertainty. The number of samples needed to achieve a certain level of certainty in a specific variable can be calculated using standard statistical techniques. These reductions in uncertainty can be translated into monetary value through reservoir simulation.

Petrophysicists can implement many advanced techniques to estimate, or to aid in the estimation of water saturation. However, very few petrophysicists know the quality of their estimates before they begin these procedures. Advanced techniques are of little value unless they can be used to reduce the uncertainty in water saturation. Simply producing more estimates of water saturation is of no value. An important goal of any analysis is to estimate water saturation and understand the level of uncertainty associated with that estimate and reduce that uncertainty as much as possible with the least cost and resources.

The preceding discussion was only concerned with the Dual Water Model. However, an analytic solution can be developed for any shaly sand model using implicit differentiation and the general formula for error propagation. Solutions have already been developed for the Archie Equation (Chen and Fang, 1986) and the Waxman-Smits Model (Freedman and Ausburn, 1985). Understanding the uncertainty in water saturation is only one of the many parameters that must be quantified. Because there is uncertainty in water saturation, there is also uncertainty in hydrocarbon pore volume which is more complicated to quantify. Defining and deriving the uncertainty in pay is probably the most significant problem. A number of methods are currently under development but need to be validated with Monte Carlo simulations.

References Cited

Bouchard, A.J, and Fox, M.J., 1999, Incorporating core analysis data uncertainty in asset value assessment: Proceedings, 1999 International Symposium of the Society of Core Analysts, Colorado School of Mines, Golden, Colorado, August 1-4, 1999, Paper 9905.

Clavier, C., Coates, G., and Dumanoir, J., 1984, Theoretical and experimental bases for the dual-water model for interpretation of shaly sands: Society of Petroleum Engineers Journal, v.24, n.2, p.153-168.

Chen, H.C., and Fang, J.H., 1986, Sensitivity analysis of the parameters in Archie's water saturation equation: The Log Analyst, September-October, p.39-44.

Freedman, R., and Ausburn, B.E., 1985, The Waxman-Smits equation of shaly sands: I, simple methods of solution, II error analysis: The Log Analyst, p.11-24.

Herron, M. M. and Matteson, A., 1993, Elemental composition and nuclear parameters of some common sedimentary minerals, Nuclear Geophysics, v. 7, no. 3, pp. 383-406.

Hill, H.J., and Milburn, J.D., 1956, Effect of clay and water salinity on electrochemical behavior of reservoir rocks: Transaction AIME, v.207, pp.65-72.

Johnson, R.E., and Kiokemeister, F.L., 1959, Calculus, Allyn and Baron Inc., Boston MA, 455p.

McKeon, D. and Scott, H., 1998, SNUPAR - A nuclear parameter code for nuclear geophysics applications, Nuclear Geophsyics, v. 2, no. 4, pp. 215-222.

Skelt, C. and Harrison, B., 1995, An integrated approach to saturation height analysis, Transactions of the SPWLA 36th Annual Logging Symposium, paper NNN.

Taylor, J.R., 1982, An Introduction to Error Analysis: Second Edition, University Science Books, Sausalito, California, 327p.

Voss, David, 1998, Quantitative Risk Analysis: John Wiley and Sons, New York, 328p.

About the Authors

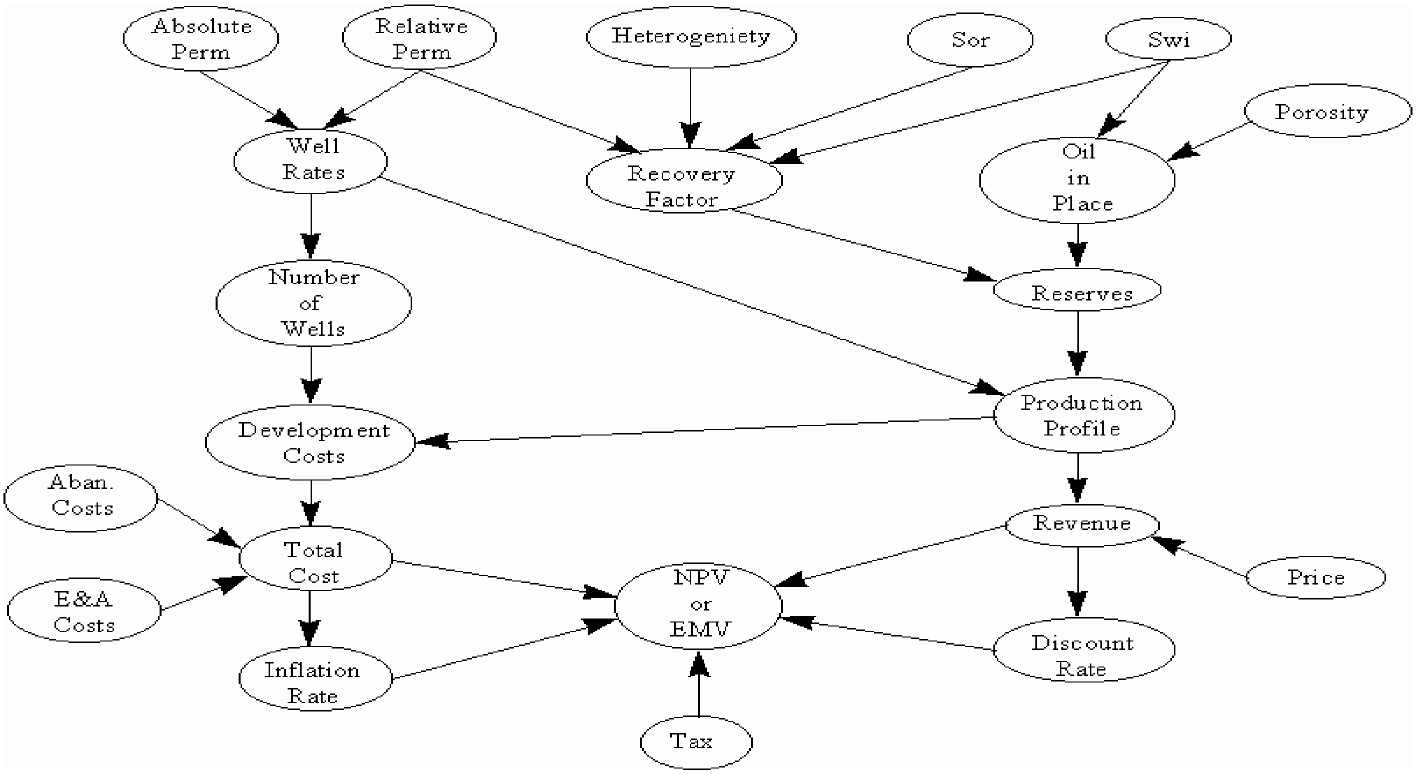
Mark Bowers received a B.S. from the University of Wisconsin-Milwaukee in 1987 an M.S. degree from Michigan Technological University in 1989 and a Ph.D. from the University of South Carolina in 1992, all in Geology. After graduation, he went to work at Conoco in a rock properties research group developing NMR interpretation techniques and models for prediction and understanding of petrophysical parameters. When petrophysics research was ended at Conoco he became responsible for petrophsycial analysis for development and exploration projects in Nigeria. Later he joined UNOCAL as a petrophyscist working the shelf and deepwater Gulf of Mexico. He is currently a petrophysicist at ExxonMobil Exploration Company primarily working exploration and appraisal wells in West Africa.

Dale Fitz received a B.S. in Chemistry from Oklahoma State in 1970 and a Ph.D. in Physical Chemistry from the University of Illinois in 1975. He has been an Alexander von Humboldt Fellow at the Max Planck Institut für Strömungsforschung in Göttingen, a Postdoctoral Fellow at the University of Toronto, and a Research and Visiting Assistant Professor at the University of Houston. He worked for 13 years at Exxon Production Research Company where he had major responsibilities for basic well logging training, cased-hole nuclear logging training, fluid contact monitoring, and nuclear tool evaluation. He worked 3 years at Esso Production Malaysia Inc. doing shaly sand petrophysical analysis and cased-hole fluid contact monitoring. He is currently a Geological Advisor with ExxonMobil Exploration Company doing cased-hole log interpretation, petrophysical field studies, and advising in petrophysical analysis for new exploration ventures. He has been an Associate Editor for the Log Analyst and is currently a Formation Evaluation Review Chairman for the SPE.

top

Figure 1 - Influence diagram illustrating the factors important for the calculation of net present value.

Many of these parameters are derived from logs or cores. After Bouchard and Fox (1999).



top

Appendix 1 - Partial Derivatives

$$\frac{\partial f}{\partial \phi_{sh}} = \phi_t^{m-1} Sw^{n-1} Vsh \left(Cw_b - Cw_f\right)$$

$$\frac{\partial f}{\partial m} = \ln \phi_t \, \phi_t^{m} \left(Sw^{n} Cw_f + Sw^{n-1} Cx\right)$$

$$\frac{\partial f}{\partial n} = \phi_t^{m} \ln Sw \left(Sw^{n} Cw_f + Sw^{n-1} Cx\right)$$

$$\frac{\partial f}{\partial Cw_f} = \phi_t^{m} Sw^{n} - \phi_t^{m-1} Sw^{n-1} Vsh \, \phi_{sh}$$

$$\frac{\partial f}{\partial Cw_b} = \phi_t^{m-1} Sw^{n-1} Vsh \, \phi_{sh}$$

$$\frac{\partial f}{\partial Vsh} = \phi_t^{m-1} Sw^{n-1} \phi_{sh} \left(Cw_b - Cw_f\right)$$

$$\frac{\partial f}{\partial a} = -C_t$$

$$\frac{\partial f}{\partial C_t} = -n$$

$$\frac{\partial f}{\partial \phi_t} = \phi_t^{m-1} \left[m Sw^{n} Cw_f + (m-1) Sw^{n-1} Cx\right]$$

$$\frac{\partial f}{\partial Sw_t} = \phi_t^{m} \left[n Sw_t Cw_f + (n-1) Sw^{n-2} Cx\right]$$

Figure 2 - Distributions of water saturation produced from the Dual Water Model and Monte Carlo distributions.

Both normal and uniform distributions were used to characterize the uncertainty of the parameters

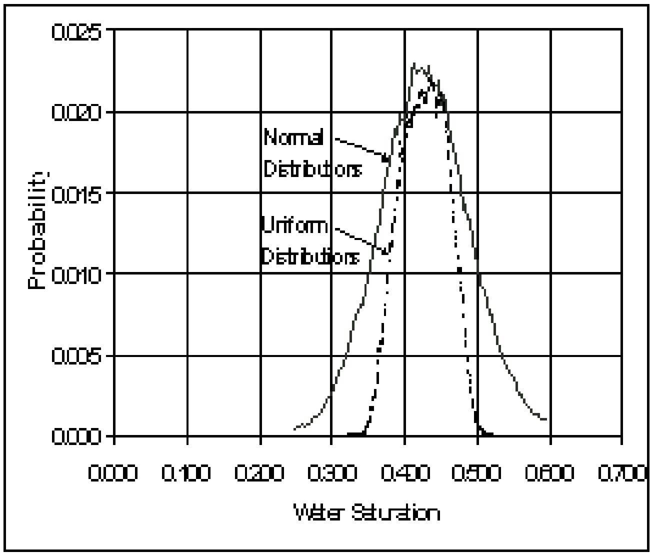
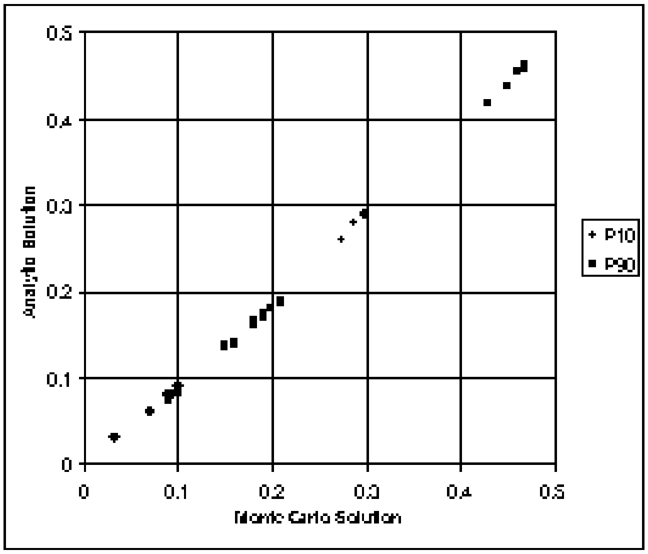


Figure 3 - Comparison of P10 and P90 estimates of water saturation derived from analytic solutions and Monte Carlo simulations

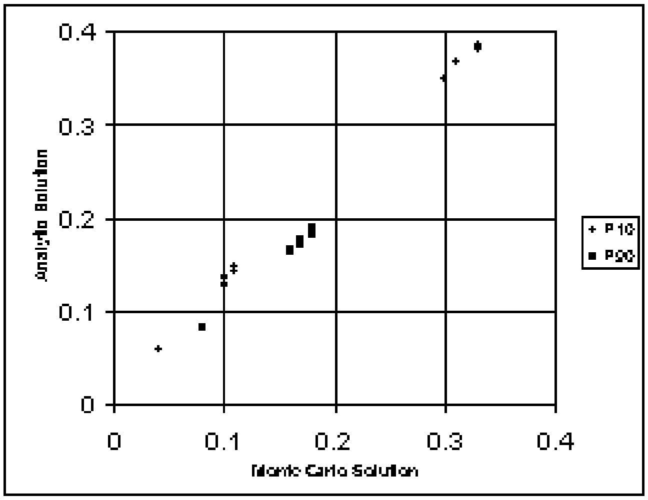
Normal distributions were used to characterize the uncertainty of the parameters for both methods.



top

Figure 4 - Comparison of P10 and P90 estimates of water saturation derived from analytic solutions and Monte Carlo simulations

Normal distributions were used to characterize the uncertainty of the parameters for the analytic solutions and uniform distributions were used to characterize the parameter for the Monte Carlo solutions.

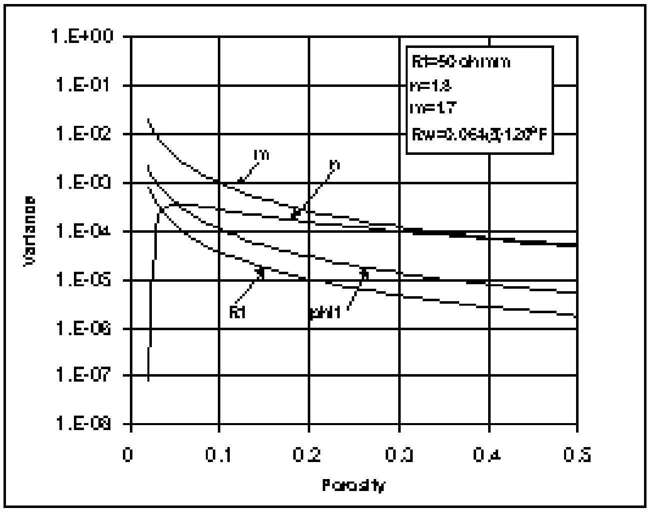


top

DiaLog

Figure 5 - Sensitivity of calculated water saturation relative the electrical properties, formation resistivity and total resistivity

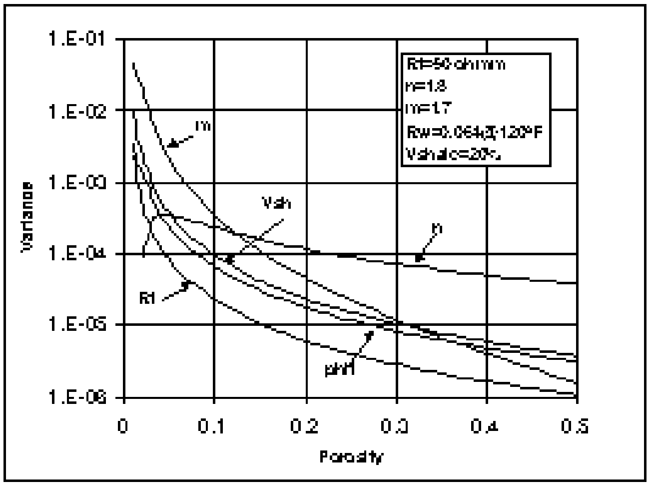
Variances were calculated using the analytic solutions derived by Chen and Fang (1986).



top

Figure 6 - Sensitivity of Calculated water saturation from DWM relative to electrical properties, formation resistivity and total porosity

Variances were calculated using analytic solutions.



top

DiaLog

Appendix 2 - Synthetic Formation

A 150-foot thick synthetic formation was developed to represent typical fluvial to marginal marine shaly sand. The synthetic formation contains two clean sands, a coarsening upward sand, four shale beds, and a calcareous tight streak. The formation was designed to have average total porosities of 28 p.u. and 17 p.u. in the cleanest sands and the shales. The shales were assigned approximately 60 volume percent clay minerals distributed initially as 55% illite, 5.5%smectite, 3% chlorite, and 36.5% kaolinite. Small amounts of siderite, orthoclase, and albite were also added with the remainder being quartz. The actual values of total porosity and each mineral at 0.5 foot intervals were varied statistically using random numbers chosen from a Gaussian distribution to make the formation look more realistic.

Permeability was assigned from the following hypothetical distribution:

$$k = 10^{[-6+\eta_k+0.3(\phi_t-.104\,V_{il}-.425\,V_{sm}-.101\,V_{ch}-.058\,V_{ka})]}$$

where h k and f t are random numbers with standard deviations of 0.4 and 0.5 respectively, and total porosity and clay mineral volumes are in pu. Water saturation was calculated from the following capillary pressure model $S_w = 100 - (100 - S_{wi}) \cdot \exp[-\{a_2(p_c - p_{th})^{-1}\}^{a_3}]$

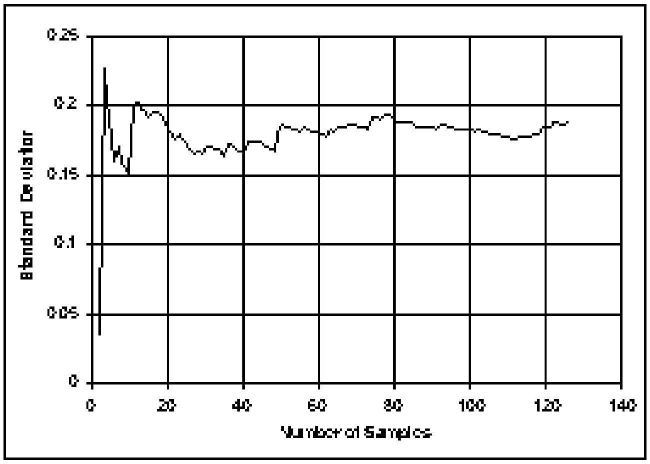
using the following hypothetical parameters: pc = 0.16(5150-d) in psi, log(Swi) = - (1.85 log(k) + 1.44), pth = k- 0.5, a2 = 0.5 + k- 1, a3 = 1 + 0.125 [log(k) - 1], and d = depth in feet. Although this is a synthetic formation, its porosity, clay volume, permeability, and water saturation are similar to real fluvial to marginal marine formations

Density and neutron log responses were calculated using SNUPAR [McKeon and Scott, 1998]. Mineral elemental concentrations and densities were based on literature values [Herron and Matteson, 1993]. Resistivity was based on the Dual Water model (Skelt and Harrison, 1995) Log responses were forward modeled assuming no invasion and using simple filters to mimic tool response.

top

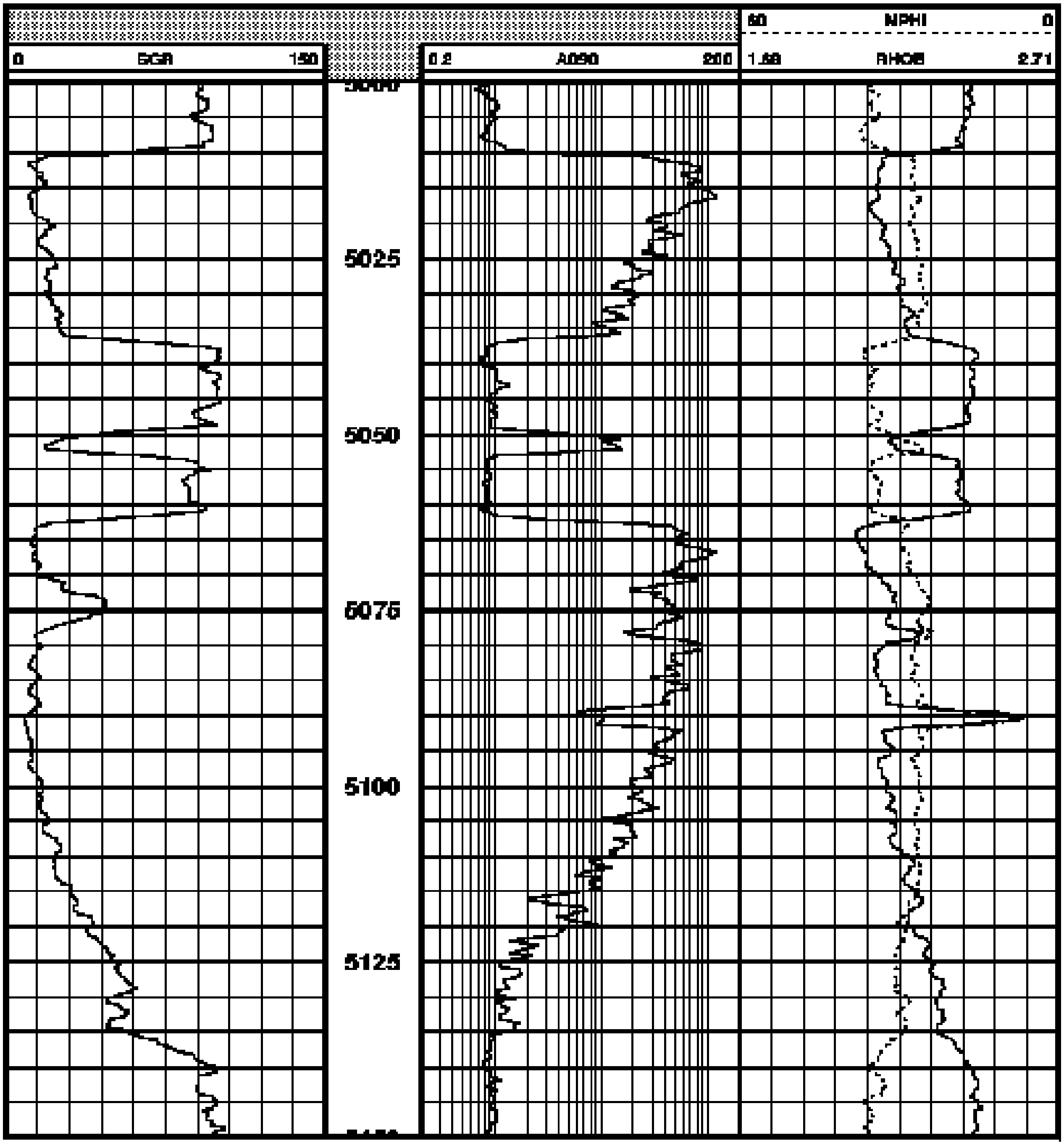
Figure 9 - The standard deviation of the cementation exponent was calculated as a function of the number of samples

The samples were randomly ordered and the standard deviation was recalculated many times, each time adding a new sample. These results also illustrate the difficulty in estimating the uncertainty in "m" and probably "n".



top

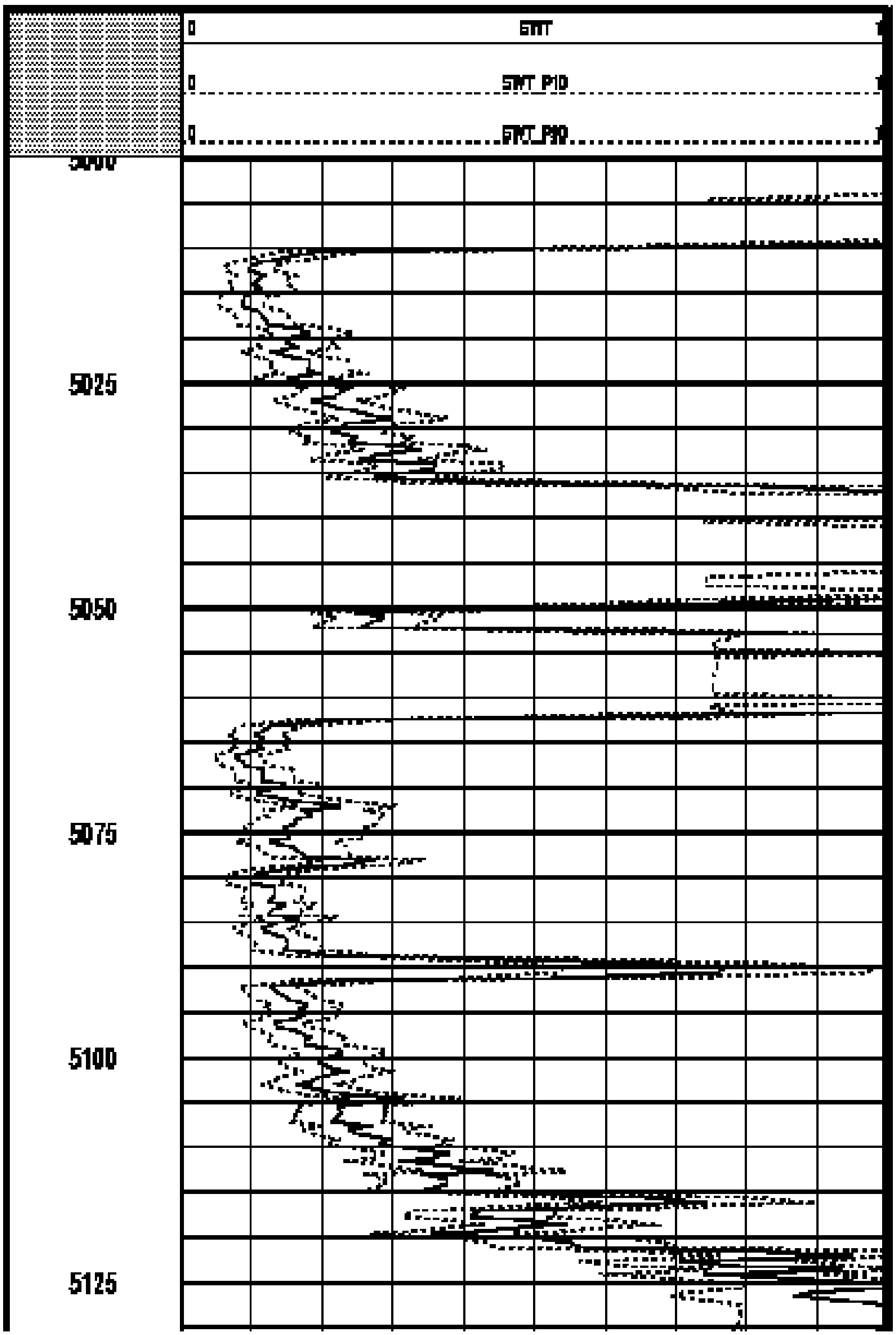
Figure 7 - Gamma ray, resistivity, neutron and density curves associated with the synthetic data set



top

Figure 8 - Probabilistic solution derived for the synthetic data set

The P50 (Swt) curve is the solid line and the P10 (Swt_P10) and P90 (Swt_P90) curves are the dashed lines.



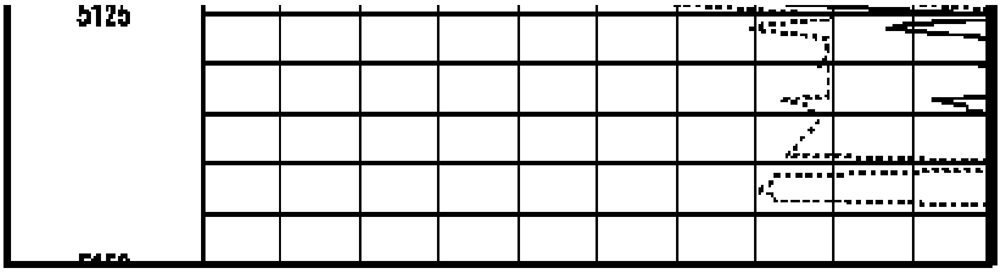
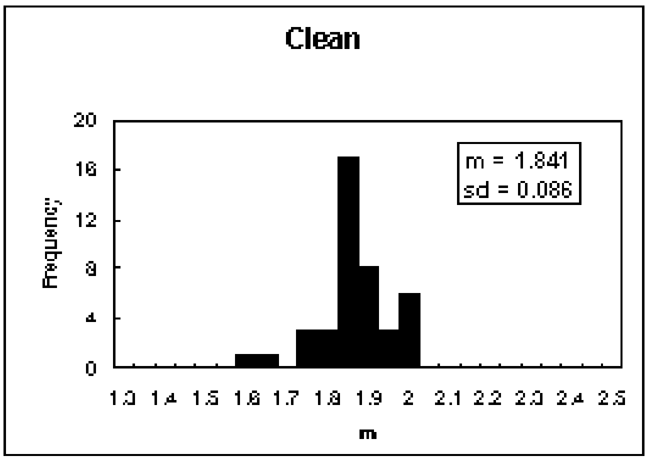

5125

Figure 10 - Histogram for cementation exponents compiled by Hill and Milburn (1956) for a clean sandstone

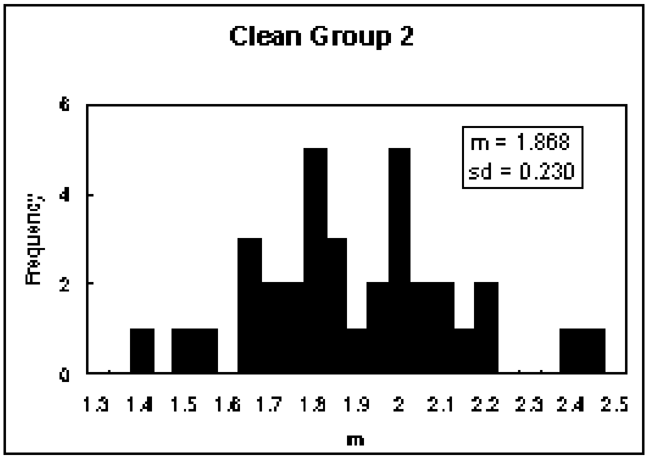
The standard deviation in "m" is relatively small compared to the standard deviation in "m" for Clean Group 2 (See Figure 11). Data compiled by Hill and Milburn (1956).



top

Figure 11 - Histogram for cementation exponents compiled by Hill and Milburn (1956) for clean sandstones (Clean Group 2)

The standard deviation in "m" is relatively large compared to the Clean sandstone data set (Figure 10) also compiled by Hill and Milburn (1956). A comparison of Figures 10 and 11 illustrate the difficulty in predicting the range in uncertainty in "m" and probably "n" even for clean sandstones.



top